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Temperature fluctuations of discrete particles in a homogeneous turbulent flow: a Lagrangian model

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Abstract

Within the frame of Lagrangian approaches for the prediction of heat transfer in dispersed two-phase flows, a new dispersion model is proposed which involves correlated stochastic processes to predict the velocity and temperature of a discrete particle along its path in terms of the instantaneous velocity and temperature of the surrounding fluid element. The dispersion problem is carefully addressed in taking into account the anisotropy of the flow and the turbulent heat flux resulting from velocity–temperature correlations. The model is used to simulate the behavior of particles suspended in a homogeneous turbulent shear flow. The numerically predicted correlations between the fluctuating quantities are in perfect agreement with the results of an analytical study by Zaichik (Phys. Fluids 11 (1999) 1521–1534). A supplementary investigation of the associated effects of non-linear drag and heat transfer is then proposed.

Keywords: Two-phase flow; Turbulence; Particles; Lagrangian simulation; Heat transfer

1. Introduction

Non-isothermal turbulent flows with solid particles or droplets are encountered in many industrial processes such as pneumatic conveying, powder drying systems or combustion. When using the so-called Eulerian–Lagrangian approach to predict such dispersed two-phase flows, the difficulty is to simulate as accurately as possible the turbulent fluctuations of the fluid and their influence on the motion and temperature of the suspended particles. In fact, it can be observed that the effect of temperature fluctuations is rarely carefully dealt with in gas–solid flows. Provided this requirement is satisfied, Lagrangian methods may improve the formulation of the closure assumptions which are needed to predict heat transfer in turbulent gas–solid flows by means of two-fluid models. For instance, the particle turbulent heat flux components $\langle u'_{pi}\theta'_{p}\rangle$, which are generally modelled by introducing a particle turbulent Prandtl number (Han et al., 1991; Kouzoubov et al., 1997), might be more accurately assessed by using data provided by Lagrangian investigations.

As regards the question of particle dynamics, the main problem in developing dispersion models is to numerically simulate the instantaneous velocity of the fluid surrounding the discrete particle (the fluid *seen* by the particle). After several pioneering works dedicated to particle dispersion in isotropic turbulence (e.g., Gosman and Ioannides, 1981; Ormancey and Martinon, 1984; Wang and Stock, 1993), Lagrangian methods for anisotropic turbulence have been suggested by Berlemont et al. (1990), Zhou and Leschziner (1991, 1997) and Burry and Bergeles (1993). Unlike such techniques, which involve a temporal step followed by a spatial step to predict the instantaneous velocity of the fluid seen, the model considered herein lies on a single first-order stochastic process according to the integral time scale T^* of the fluid seen by the discrete particle. The reader is referred to Wang and Stock (1993) or Pozorski and Minier (1998) who have proposed analytical expressions of T^* in terms of particle inertia and mean drift velocity, and to Pétrissans et al. (2000) for additional information about the capabilities of the dispersion model, which can account for the inertia effect, the continuity effect and the crossing trajectories effect, as well as for the so-called spurious drift in case of non-homogeneous turbulence (Legg and Raupach, 1982).

As concerns heat transfer prediction, it is worth mentioning the Lagrangian simulation of non-isothermal gas–solid flow by Avila and Cervantes (1995), who have used an eddy-interaction model by assuming equal dynamic and thermal integral time scales of the fluid. However, the requirement that the fluctuations must be connected by the given velocity–temperature correlations of the fluid $\langle u'_{fi}\theta'_{f}\rangle$ was not taken into account in their model. Heat transfer predictions in a turbulent gas–solid pipe flow using an enhanced Lagrangian model were recently presented by Moissette et al. (2000). The main objective was to improve the description of the effect of

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Notation

a, b_{ik}, c_k	coefficients used to generate correlated random variables
C_D	drag coefficient
c_p	particle specific heat
d_p	particle diameter
G_u	mean velocity gradient
G_θ	mean temperature gradient
h_p	particle-to-fluid heat transfer coefficient
k	fluid turbulent kinetic energy
m_p	particle mass
Nu_p	particle Nusselt number
Pr	Prandtl number
Re_k	particle Reynolds number based on $\sqrt{k}$
Re_p	particle Reynolds number
S_u	dimensionless mean velocity gradient
S_θ	dimensionless mean temperature gradient
T^*	integral time scale
T^*_θ	temperature integral time scale
t	time
$\mathbf{U}_f$	fluid velocity vector
$\mathbf{U}_p$	particle velocity vector
u_{fi}	fluid velocity component
u_{pi}	particle velocity component
$\mathbf{X}_p$	particle position vector
x	streamwise direction
y	mean shear direction
z	spanwise direction

Greeks

Δt	time step
θ_f	fluid temperature
θ_p	particle temperature
ν_f	fluid kinematic viscosity
ξ	temperature random disturbance
ρ_f	fluid density
ρ_p	particle density
τ_p	particle dynamic relaxation time
τ_{p0}	Stokesian particle dynamic relaxation time
τ_{pe}	effective dynamic relaxation time
$\tau_{p\theta}$	particle thermal relaxation time
$\tau_{p\theta 0}$	particle thermal relaxation time for $Nu_p = 2$
$\tau_{p\theta e}$	effective thermal relaxation time
χ_i, ζ	independent Gaussian variables
ψ_i	velocity random disturbance
Ω_u	dynamic Stokes number
Ω_{ue}	modified dynamic Stokes number
Ω_θ	thermal Stokes number

Subscripts and superscripts

$\langle\ \rangle$	ensemble average
$'$	fluctuating quantity
n	quantity at time $n\Delta t$

turbulence on particles, but also to get as accurate information as possible about the particle turbulent heat flux components $\langle u'_{pi}\theta'_p\rangle$. For this purpose, the instantaneous velocity and temperature fluctuations of the fluid seen by the particles are simulated by means of specific stochastic processes which make them satisfy the local value of $\langle u'_{fi}\theta'_f\rangle$.

The present work is devoted to the validation of such a new Lagrangian model for heat transfer prediction. The case dealt with herein is a homogeneous turbulent flow with constant gradients of mean velocity and temperature. Analytical expressions of the second-order moments have been given in this case by Zaichik (1999), according to a theoretical analysis in the frame of a p.d.f. approach (Zaichik, 1999; Zaichik et al., 1997), with the assumption that the mean velocity and temperature of particles are equal to the fluid ones (no external force or heat source). The model is first thoroughly described, then numerical results are given and compared to the analytical ones. Finally, supplementary calculations are devoted to the investigation of possible non-linear drag and heat transfer effects.

2. Formulation

2.1. Overview of the Lagrangian simulation

Particles are tracked in a homogeneous shear flow with constant mean temperature gradient by solving the equation of motion under the assumptions of incompressible fluid and dilute suspension. Hence, inter-particle collisions are neglected, as well as the effect of particles upon the fluid flow (one-way simulation). Moreover, spherical solid particles are considered, and the only force acting on particles is the drag force (no gravity).

The dynamic equations are reduced to

$$\frac{\mathrm{d}\mathbf{X}_p}{\mathrm{d}t} = \mathbf{U}_p, \quad \frac{\mathrm{d}\mathbf{U}_p}{\mathrm{d}t} = \frac{\mathbf{U}_f - \mathbf{U}_p}{\tau_p}, \tag{1}$$

where $\mathbf{X}_p$ and $\mathbf{U}_p$ are the position and velocity of the particle, $\mathbf{U}_f$ is the instantaneous local fluid velocity and

$$\tau_p = \frac{4}{3}\rho_p d_p/(\rho_f C_D\|\mathbf{U}_f - \mathbf{U}_p\|)$$

is the relaxation time of the particle: τ_p is constant if Stokes' drag law applies ($Re_p \ll 1, Re_p$ standing for the instantaneous particle Reynolds number), otherwise τ_p depends on Re_p (non-linear drag force, see Section 2.3).

In addition, the temperature of each particle is computed along its trajectory assuming that each particle has a uniform temperature and that the temperature is sufficiently low to neglect radiative transfer. Under such assumptions, the energy balance takes the following form:

$$\frac{\mathrm{d}\theta_p}{\mathrm{d}t} = \frac{\theta_f - \theta_p}{\tau_{p\theta}}, \tag{2}$$

where θ_p and θ_f are the instantaneous temperatures of the particle and of the fluid seen, respectively, and $\tau_{p\theta}$ is the particle thermal relaxation time, defined by

$$\tau_{p\theta} = \frac{m_p c_p}{\pi d_p^2 h_p} \tag{3}$$

which is either constant or varies as a function of Re_p, depending on the particle Nusselt number expression used to estimate h_p (see Section 2.3).

In the following sections, the fluctuating parts of the fluid velocity and temperature at location $\mathbf{X}(t)$ and time t are denoted by $\mathbf{u}'_f(\mathbf{X}, t)$ and $\theta'_f(\mathbf{X}, t)$, respectively.

2.2. Dispersion modelling

The instantaneous velocity and temperature fluctuations of the fluid at each point of the discrete particle trajectory are simulated by means of appropriate stochastic processes. Using any available dispersion model to predict the velocity fluctuations of the fluid at the discrete particle location $\mathbf{X}_p(t)$, the following first-order autoregressive process is used to generate the fluid temperature fluctuation at time t and location $\mathbf{X}_p(t)$:

$$\theta'_n = \theta'_{n-1} \exp\left(-\frac{\Delta t}{T^*_\theta}\right) + \xi_n, \tag{4}$$

where θ'_n stands for $\theta'_f(\mathbf{X}_p(n\Delta t), n\Delta t)$, T^*_θ is the temperature integral time scale of the fluid seen by the particle, and the ξ_n are Gaussian variables with zero mean value and with variance given by

$$\langle \xi_n^2 \rangle = \left(1 - \exp\left(-2\frac{\Delta t}{T^*_\theta}\right)\right)\langle \theta'^2_f \rangle \tag{5}$$

from stationarity requirements under quasi-homogeneity conditions. In case of homogeneous turbulence, such a first-order time series is consistent with an exponentially decaying time correlation of the temperature fluctuation $\theta'_f(\mathbf{X}_p(t), t)$ along the particle path.

In order to ensure consistency of the generated temperature fluctuations with the required values of the velocity–temperature one-point correlations $\langle u'_{fi}\theta'_f \rangle$ of the fluid, the ξ_n have to be linked with the random terms appearing in the process used to generate the velocity fluctuation. Let us examine the case where the velocity fluctuations of the fluid seen by the particle obey the same kind of first-order stochastic process (consistent with an exponentially decaying time correlation of the velocity fluctuation). Introducing $u'_{i_n} = u'_{fi}(\mathbf{X}_p(n\Delta t), n\Delta t)$ and three Gaussian variables ψ_{i_n}, and denoting by T^*_i the integral time scales of the fluid seen, the corresponding time series can be written as

$$u'_{i_n} = u'_{i_{n-1}} \exp\left(-\frac{\Delta t}{T^*_i}\right) + \psi_{i_n} \tag{6}$$

showing that ξ_n must satisfy

$$\langle \xi_n \psi_{i_n} \rangle = \left[1 - \exp\left(-\Delta t\left(\frac{1}{T^*_\theta} + \frac{1}{T^*_i}\right)\right)\right]\langle u'_{fi}\theta'_f \rangle. \tag{7}$$

Moreover, anisotropy can be introduced into the stochastic process (6) by linking the white noise disturbances ψ_{i_n} in order to satisfy the prescribed values of the Reynolds stresses. From Eq. (6), we get the conditions to be fulfilled by the covariances:

$$\langle \psi_{i_n} \psi_{jn} \rangle = \left[1 - \exp\left(-\Delta t\left(\frac{1}{T^*_i} + \frac{1}{T^*_j}\right)\right)\right]\langle u'_{fi}u'_{fj} \rangle. \tag{8}$$

A simple procedure to generate such correlated random numbers is to consider a set of three independent random variables χ_i selected from a normal p.d.f. with zero mean and variance unity, and to build ψ_i as follows:

$$\psi_i = b_{ik}\chi_k, \tag{9}$$

where the coefficients b_{ik} are calculated, without loss of generality, according to

$$\begin{aligned} &b_{11} = \langle \psi_1^2 \rangle^{1/2}, \quad b_{12} = 0, \quad b_{13} = 0, \\ &b_{i1} = b_{11}^{-1}\langle \psi_i \psi_1 \rangle, \\ &b_{i2}b_{22} = \langle \psi_i \psi_2 \rangle - b_{21}b_{i1}, \quad i = 2,3, \\ &b_{i3}b_{33} = \langle \psi_i \psi_3 \rangle - b_{31}b_{i1} - b_{32}b_{i2}, \quad i = 2,3. \end{aligned} \tag{10}$$

The conditions specified by Eqs. (5) and (7) can easily be satisfied by selecting a random number ζ from a normal p.d.f. with zero mean and variance unity, and building ξ by

$$\xi = a\zeta + c_k\psi_k, \tag{11}$$

where the coefficients a and c_k are obtained from the conditions on $\langle \xi^2 \rangle$ and $\langle \xi\psi_i \rangle$ according to Eqs. (5) and (7):

$$c_k\langle \psi_k \psi_i \rangle = \langle \xi\psi_i \rangle, \quad a^2 = \langle \xi^2 \rangle - \left\langle (c_k\psi_k)^2 \right\rangle. \tag{12}$$

In the present study, which aims at improving our dispersion model with regard to heat transfer, this method is applied to a homogeneous shear flow by taking into account the fluid Reynolds stress $\langle u'_{fx}u'_{fy} \rangle$ and the fluid velocity–temperature correlations $\langle u'_{fx}\theta'_f \rangle$, $\langle u'_{fy}\theta'_f \rangle$ (flow direction x, shear direction y).

2.3. Numerical conditions

In order to compare the results of the proposed dispersion model with the analytical solution derived by Zaichik (1999), the flow characteristics are evaluated from the experimental data of Tavoularis and Corrsin (1981) obtained in a nearly homogeneous shear flow with constant mean gradients of velocity and temperature:

$$\begin{aligned} &\frac{\langle u'^2_{fx} \rangle}{k} = 1.07, \quad \frac{\langle u'^2_{fy} \rangle}{k} = 0.37, \\ &\frac{\langle u'^2_{fz} \rangle}{k} = 0.56, \quad \frac{\langle u'_{fx}u'_{fy} \rangle}{\langle u'^2_{fx} \rangle^{1/2}\langle u'^2_{fy} \rangle^{1/2}} = -0.45, \\ &\frac{\langle u'_{fx}\theta'_f \rangle}{\langle u'^2_{fx} \rangle^{1/2}\langle \theta'^2_f \rangle^{1/2}} = 0.59, \quad \frac{\langle u'_{fy}\theta'_f \rangle}{\langle u'^2_{fy} \rangle^{1/2}\langle \theta'^2_f \rangle^{1/2}} = -0.45, \end{aligned} \tag{13}$$

where $k = \langle u'_{fj}u'_{fj} \rangle/2$ is the turbulent kinetic energy of the fluid. According to the assumptions made by Zaichik (1999) in his quasi-state solution for a homogeneous layer, the velocity and temperature integral time scales of the fluid seen by the particles are assumed to be constant and equal to each other, i.e., $T^*_i = T^*_\theta = T^*$. The velocity gradient parameter $S_u = T^*G_u$, where G_u is the mean velocity gradient, is assumed to be equal to the temperature gradient parameter $S_\theta = (T^*k^{0.5}/\langle \theta'^2_f \rangle^{0.5})G_\theta$, where G_θ is the mean temperature gradient. The dimensionless time parameters (similar to Stokes numbers) Ω_u and Ω_θ are defined by $\Omega_u = \tau_p/T^*$and $\Omega_\theta = \tau_{p\theta}/T^*_\theta$.

Numerical calculations are based on the computation of a large number of particle trajectories through the flow (2 000 000 particles are injected). Averaging is performed to compute statistical quantities such as the particle velocity–temperature correlations. Initially, particles are uniformly distributed along a straight line with given length L, in the direction of the flow gradients (Fig. 1). In order to avoid any boundary effect, only particles with final position y lying within the small segment $[-L/20, +L/20]$ are taken into account to evaluate the statistical quantities. This small test section, one tenth of the injection length L, is divided into 20 cells of width Δy. For each test, at least 1000 particles have

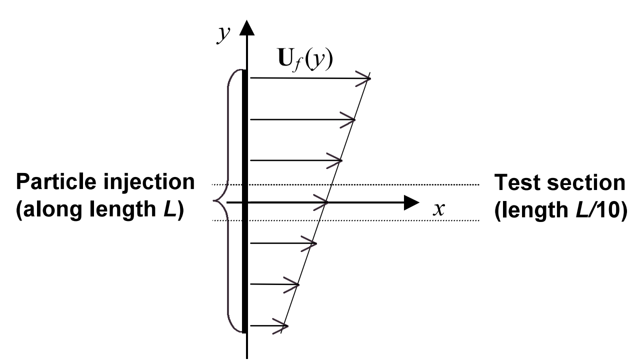

Fig. 1. Sketch of the test configuration.

been observed in each cell of the test section at the end of the computation. Particle trajectories and temperature are computed by integrating Eqs. (1) and (2) by means of a first-order Runge–Kutta method, the fluid fluctuating quantities being calculated according to the above-presented dispersion model. For the sake of reliability, a small enough time step is chosen, namely $\Delta t = \min(T^*/10, \tau_p/10)$, and the total tracking time is not less than $\max(5T^*, 5\tau_p)$.

First, computations have been carried out with the assumption that the particle relaxation times τ_p and $\tau_{p\theta}$ are constant (i.e., $Re \ll 1$ and $Nu_p = 2$) and equal to each other. Since the velocity and temperature integral time scales are also equal to each other, the dynamic and thermal Stokes numbers Ω_u and Ω_θ are identical. For each value of the gradient parameter, various tests have been performed for several values of the particle relaxation times $\tau_p = \tau_{p\theta}$, leading to Stokes numbers $\Omega_u = \Omega_\theta$ varying between 0 and 10. The turbulent kinetic energy k and the integral time scale T^* do not affect the results since all quantities are non-dimensional, therefore they are fixed to arbitrary values.

Further computations have been performed in the case of non-linear drag force and heat transfer. In this case, the dynamic and thermal particle relaxation times do not remain constant along the particle trajectory, therefore they have to be computed at each time step. The relaxation times τ_p and $\tau_{p\theta}$ depend, respectively, on the particle drag coefficient C_D, evaluated from the correlation of Morsi and Alexander (1972), and on the particle Nusselt number Nu_p, estimated from the following commonly used correlation: $Nu_p = 2 + 0.6Re_p^{0.5}Pr^{0.33}$, where Pr is the fluid Prandtl number. Now, besides the dimensionless gradient parameters S_u, S_θ and the Stokes numbers Ω_u, Ω_θ, another nondimensional parameter has to be considered, namely $Re_k = \sqrt{k} \cdot d_p/\nu_f$. This is because the possible non-linear drag and heat transfer effects are depending on the particle Reynolds number Re_p based on the instantaneous relative velocity, which is not known before performing the simulation. Since there is no mean drift velocity here (no gravity force), the only characteristic Reynolds number which can be fixed is Re_k, keeping in mind that the effective average particle Reynolds number will depend on both Re_k and Ω_u. For small inertia particles, which follow rather well the fluid fluctuations, the average relative velocity may be expected to be small compared to $\sqrt{k}$, therefore $\langle Re_p \rangle \ll Re_k$, whereas we may expect $\langle Re_p \rangle \approx Re_k$ for very

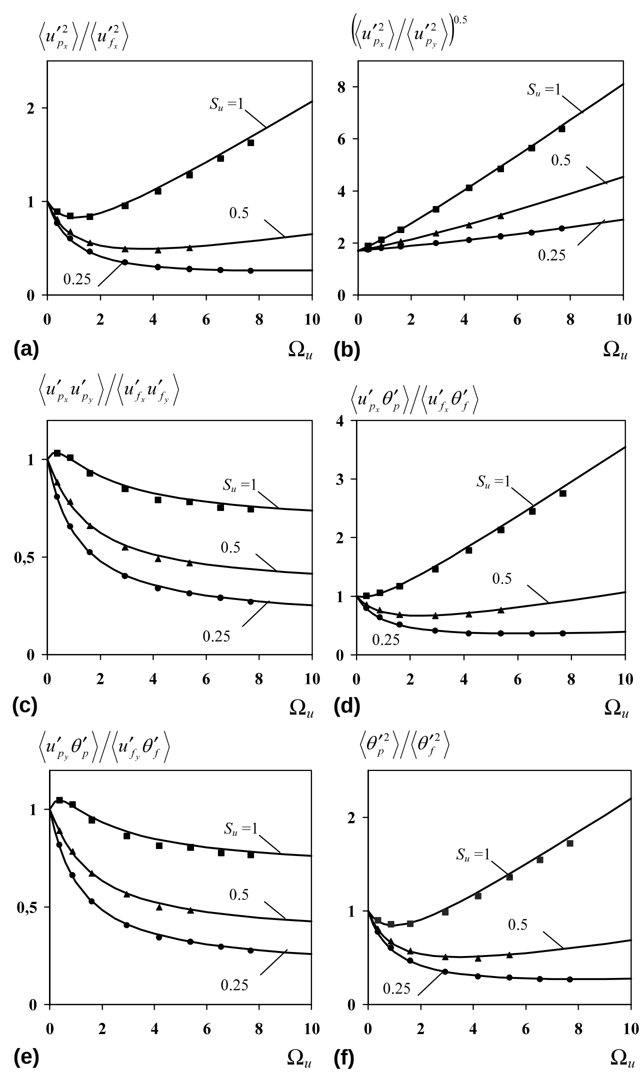

Fig. 2. Particle kinetic stresses (a)–(c), heat fluxes (d)–(e) and temperature fluctuation intensity (f). Solid curves are for Zaichik's analytical results and symbols are for the present computation.

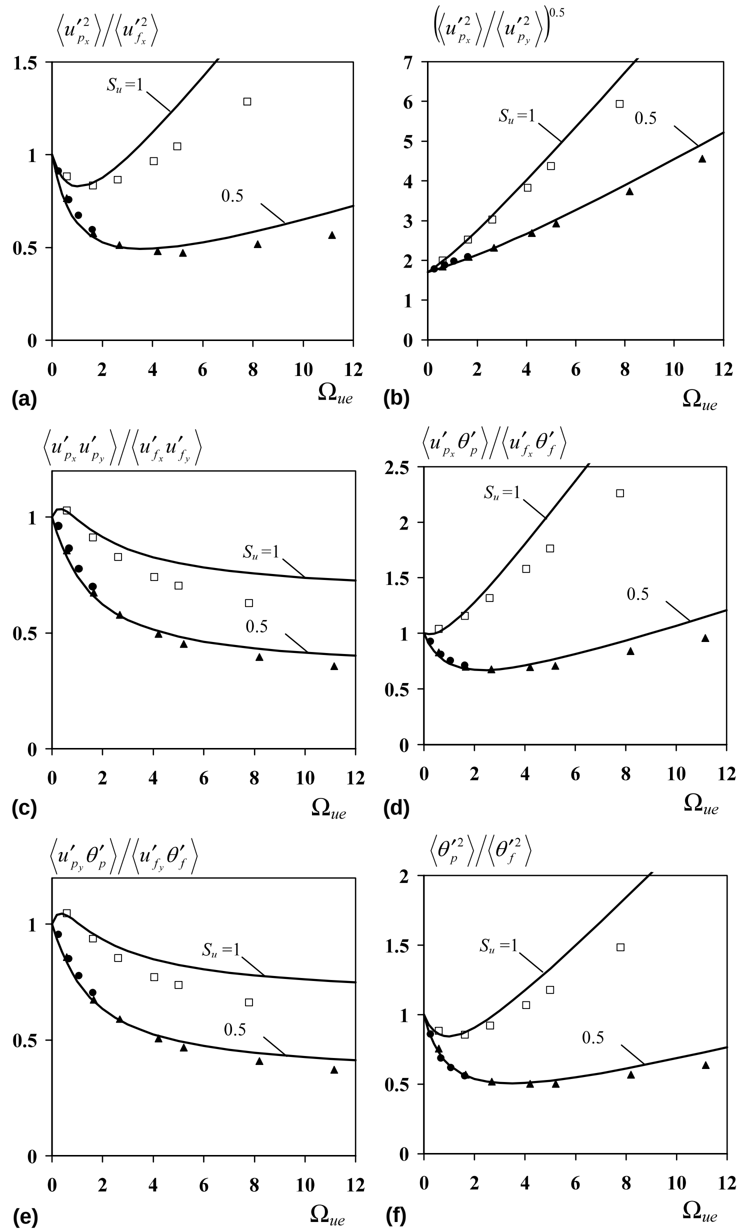

Fig. 3. Particle kinetic stresses (a)–(c), heat fluxes (d)–(e) and temperature fluctuation intensity (f) in case of non-linear drag and heat transfer effects. Solid curves are for Zaichik's analytical results and symbols are for the present computation. (▲) $Re_k = 10$, $S_u = S_\theta = 0.5$; (●) $Re_k = 100$, $S_u = S_\theta = 0.5$; (□) $Re_k = 10$, $S_u = S_\theta = 1$.

heavy particles. The flow characteristics are the same as for the linear drag case. However, the assumption $\Omega_u = \Omega_\theta$ is now based on the Stokesian particle relaxation time τ_{p0}, since the instantaneous particle relaxation times are not constant along the particle path. Assuming, as above, $S_u = S_\theta$, the influence of Re_k can be investigated by comparing the numerical predictions with the results obtained under the assumption of linear drag. Computations have been carried out for $S_u = S_\theta = 0.5$ and 1, $Re_k = 10$ and 100, and $\Omega_u = \Omega_\theta$ ranging from 1 to 10.

3. Numerical results

Figs. 2 and 3 display the particle kinetic stresses (plots (a), (b) and (c)), turbulent heat fluxes (plots (d) and (e)) and temperature fluctuations (plot (f)) as a function of the particle Stokes number Ω_u for various values of the gradient parameter S_u. In Fig. 2, comparison is made between Zaichik's analytical results (solid curves) and the results from the present Lagrangian model, assuming constant and equal particle dynamic and thermal relaxation times, according to Zaichik's hypothesis. Fig. 2 shows that the proposed dispersion model leads to perfect agreement with the analytical solution for all particle fluctuating quantities. The main features of the fluid–solid flow, such as the development of anisotropy in particle r.m.s. velocity (Fig. 2(b)) or the similarities between the particle turbulent heat fluxes and particle kinetic stresses are correctly simulated. It may be mentioned that Zaichik's analysis also assumes exponentially decaying correlation functions for the fluid seen by the particles, therefore it is not surprising to obtain such a good agreement. For detailed analysis of the physical sense of the whole curves, one will refer to Zaichik (1999), whose theoretical investigation has been restricted to constant relaxation times, however. It may therefore be interesting to use the proposed Lagrangian technique in order to assess the effect of non-linear drag force and heat transfer, which cannot be taken into account in a rigorous way in analytical investigations.

Corresponding results are presented in Fig. 3, which illustrates the effects of non-linear drag force and Reynolds dependent Nusselt number. Since τ_p changes along the particle trajectory, the particle fluctuating quantities are plotted against the modified Stokes number Ω_{ue} based on the effective particle relaxation time, which is the average relaxation time defined by

$$\tau_{pe} = \tau_{p0} \frac{24}{C_D \langle Re_p \rangle}, \tag{14}$$

where τ_{p0} is the Stokesian particle relaxation time, and C_D is the drag coefficient at particle Reynolds number $\langle Re_p \rangle$ based on the particle average relative velocity, computed from the Lagrangian simulation as $\langle \sum_i (u'_{fi} - u'_{pi})^2 \rangle^{1/2}$. In a similar way, the effective thermal relaxation time is defined by $\tau_{p\theta e} = \tau_{p\theta 0}(2/Nu_p)$ where $\tau_{p\theta 0}$ is the particle relaxation time assuming a constant particle Nusselt number (equal to 2), and Nu_p is the Nusselt number at particle Reynolds number $\langle Re_p \rangle$. It should be mentioned, however, that $\tau_{p\theta e}$ and τ_{pe} remain close to each other in the range of $\langle Re_p \rangle$ investigated here, due to the assumption $\tau_{p\theta 0} = \tau_{p0}$. Comparison is made between the cases of linear drag force and non-linear drag force (with $Re_k = 10$ and 100), for $S_u = S_\theta = 0.5$ and 1, and τ_{p0}/T^* ranging from 1 to 10.

For particles experiencing non-linear drag and heat transfer effects, it can be concluded from the plots in Fig. 3 that the particle kinetic stresses, turbulent heat fluxes and temperature fluctuation intensity may be approximated in a satisfactory way by considering the inertia parameters (or Stokes numbers) Ω_u and Ω_θ based on effective relaxation times depending on the mean relative velocity. However, better agreement is observed for $S_u = S_\theta = 0.5$: for larger values of the gradient parameters, the particle kinetic stresses and heat fluxes are overestimated, as can be seen from the curves for $S_u = S_\theta = 1$. Therefore, the present Lagrangian calculations confirm that the use of an effective mean relaxation time, as generally done in Eulerian calculations, allows the non-linear drag and heat transfer effects to be correctly taken into account, provided that the velocity and temperature gradients are not too large.

4. Conclusion and prospects

An improved Lagrangian model for the simulation of heat transfer in turbulent particulate two-phase flows has been described and tested by comparing the numerical predictions with the analytical results in a homogeneous shear flow. The dispersion model is built in such a way that the fluid velocity and temperature fluctuations are correlated according to the prescribed Reynolds stresses and velocity–temperature correlations. The predicted particle r.m.s. velocities and temperatures, as well as particle kinetic stresses and turbulent heat fluxes, have been shown to be in perfect agreement with the theoretical results. The Lagrangian technique has been used to investigate the effects of non-linear drag and heat transfer. Extension of the theoretical expressions by using effective dynamic and thermal relaxation times has been shown to be possible in Eulerian calculations, provided that the velocity and temperature gradients are not too high.

In order to extend the proposed model for non-homogeneous turbulence, appropriate expressions of the integral time scales should be used, as described by Moissette et al. (2000) for example. Moreover, it must be kept in mind that the gradient of the fluid r.m.s. velocity must be introduced into the stochastic process used to generate the fluid velocity, as done by Pétrissans et al. (2000) in a pipe flow, in order to avoid the so-called spurious drift effect (Legg and Raupach, 1982).

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